Disciplinärt Urskiljande av Representationer i Matematiken - Vad ser studenterna och vad ser de inte?

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Outline

- Introducing the theoretical framework
    - Sociala semiotics
    - Disciplinary discernment
- Example from mathematic
- Implications
- Conclusions
- Discussion

Social Semiotics, Semiotic Resources and Representations

- **Social semiotics** - ”the study of the development and reproduction of specialized systems of meaning making in particular sections of society” (Airey & Linder 2017).

- All **communication** in a particular social group is viewed as being realized through the **use of semiotic resources.** In social semiotics the particular **meanings assigned** to these semiotic resources are **negotiated** within the **social group** itself and they have often developed over an **extended period of time**.

- **Semiotic resources:** Graphs, diagrams, sketches, figures, mathematics, gestures, specialist language, (activities, tools,) etc., often referred to as **REPRESENTATIONS or MODES**.

Differences between social semiotics and representational approach to learning

- **Social semiotics** focuses primarily on **group meaning making** - starting point: the ways in which professional mathematicians make and share meaning using semiotic resources.

- **Social semiotics** includes all forms of meaning making through semiotic resources - ***representations, tools and activities***:
  - What **meaning** can this resource convey and how is that meaning constructed by students?
  - Contrasted against: **What is** this a representation of?

- **Semiotic resources** have a range of **meaning potentials** - **disciplinary affordances**: "the agreed meaning making functions that a semiotic resource fulfills for the disciplinary community" (Airey & Linder 2017).

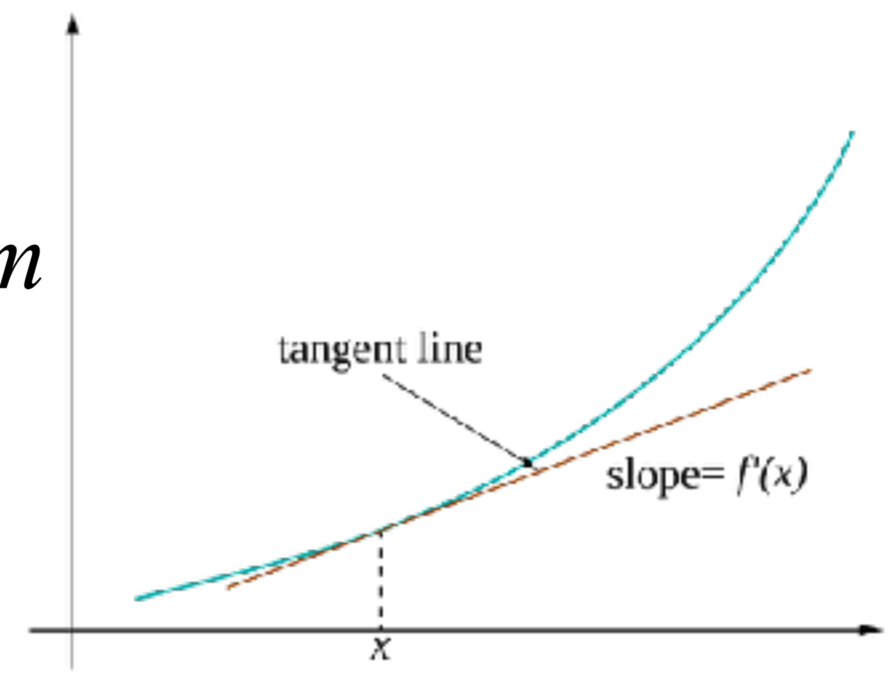
Critical constellations of semiotic resources

- **Disciplinary meaning** is usually realized through the coordination of **combinations of semiotic resources** with different disciplinary affordances.

$$y = f(x); \quad Slope = f'(x)$$

$$\text{Tangent line: } y = kx + m = x \cdot f'(x) + m$$

$$y = \frac{y_2 - y_1}{x_2 - x_1} \cdot x + m$$



- There is a **critical constellation** of disciplinary semiotic resources that is necessary for an appropriate experience of disciplinary knowledge.

Disciplinary shorthand

- **Teachers and mathematicians** only use a smaller **subset of the critical constellation** in their day-to-day work.
- In fact, in many situations only a single semiotic resource is used—an equation or a diagram say—which functions as a **disciplinary shorthand** to activate a whole concept.
- Example: Maxwells equations, derivatives, integrals, etc.

Representational competency

- Characterize the **mastering of disciplinary-specific semiotic resources**.

- If a person is said to be **fluent** in a particular semiotic resource, then they have **come to understand** the particular way(s) that **the discipline uses** that resource to **share and work with** mathematics knowledge in a given situation.

Fluency and Discourse imitation

- It is only when **fluency** in a critical constellation of semiotic resources is **combined** with an **appreciation** of the associated disciplinary affordances **that appropriate and disciplinary meaning making becomes possible**.

- **Discourse imitation.** The ability to use semiotic resources with limited or no associated disciplinary understanding.

  - Example: derivatives of polynomial; students may learn the ”rule”, be able to discuss it, but not understand why.

$$\frac{d}{dx}(x^3 + 2x - 4) = 3x^2 + 2$$

Summary so far

- In order for students to appropriately **experience disciplinary knowledge** they need to **become fluent** in the **use of each separate** semiotic resource that makes up the **critical constellation** for that particular piece of knowledge.

- However, **fluency** in the critical constellation alone is **not sufficient**.

- Students still **need to come to appreciate** the disciplinary affordance **of each of these resources** and **how they can be coordinated** before they can **understand the concept** in an appropriate, disciplinary manner.

Pedagogical and disciplinary affordances

- **Pedagogical affordance** as "the aptness of a semiotic resource for the teaching and learning of some particular educational content".
  It refers to **how useful** a given semiotic resource tends to be for teaching and learning **a specific piece of content**.

  - Example: The definition of derivatives may not be the best thing to use when finding the derivative of a polynomial in a classroom situation.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h \to 0} \frac{a^x a^h - a^x}{h} = \lim_{h \to 0} \frac{a^x(a^h - 1)}{h}$$

- **Disciplinary affordance**: "the agreed meaning making functions that a semiotic resource fulfills for a particular disciplinary community".

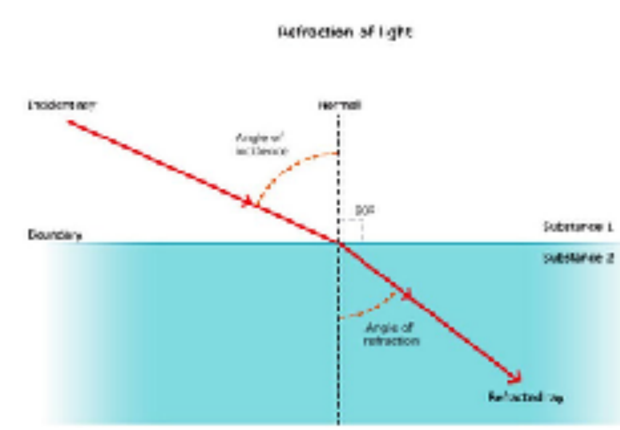
- The power of the term **for educational work** is that **learning** can now be framed as **coming to discern the disciplinary affordances of semiotic resources**.

Disciplinary relevant aspects

- In the same way that semiotic resources have a range of meaning potentials that need to be selected between, **disciplinary concepts have a range of aspects associated with them**: typically, for a given educational situation only a discrete set of these aspects will be relevant and/or needed.

  - Example:
    -System of equations
    -Refraction (physics…)

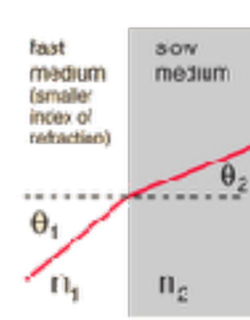
$$\begin{cases} 3x + 1 = y \\ -x - y = 2 \end{cases}$$

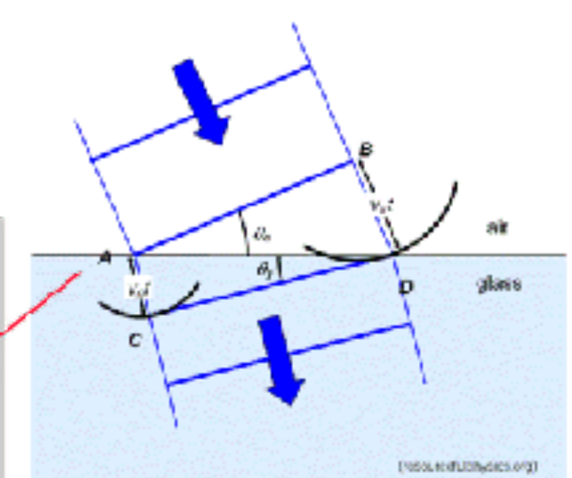


- **Definition**: Those aspects of mathematic concepts that have particular relevance for carrying out a specific task.

Snell's Law

$$\frac{n_1}{n_2} = \frac{\sin \theta_2}{\sin \theta_1}$$





Noticing disciplinary relevant aspects: The variation theory of learning

- **”Put simply, humans tend to notice that which varies.”**
- Learning can be made possible through a three step process:
    1. Identify the disciplinary relevant aspects for a given task
    2. Select appropriate semiotic resources that showcase these disciplinary relevant aspects
    3. Vary each of the aspects whilst holding everything else in the semiotic resource constant (i.e. setting up difference against a background of sameness).
- Example: Determining the slope of curves using graphical representation.

Multiple semiotic resources

- The disciplinary relevant aspects for a task are made available by combining semiotic resources ***by the teacher***.

- **The task for a teacher** becomes one of encouraging and enhancing the possibility of **disciplinary discernment *by the students***.

- This entails noticing and focusing on the appropriate disciplinary aspects across a range of semiotic resources, whilst ‘pushing’ unrelated disciplinary aspects and surface features into background awareness.

Disciplinary discernment

- **Noticing** something (Mason 2002), **reflecting** on it (Dewey 1933; Schön 1983), and **constructing meaning** from a **disciplinary perspective (Eriksson et al. 2014)**.

- But different persons discern things differently…

The Anatomy of Disciplinary Discernment (ADD)

Eriksson et al. (2014)

- The **ways** in which the disciplinary affordances of a given representation may be **discerned**.

- A **hierarchy** of **what is focused** on and **how it is interpreted** in an appropriate, disciplinary manner.

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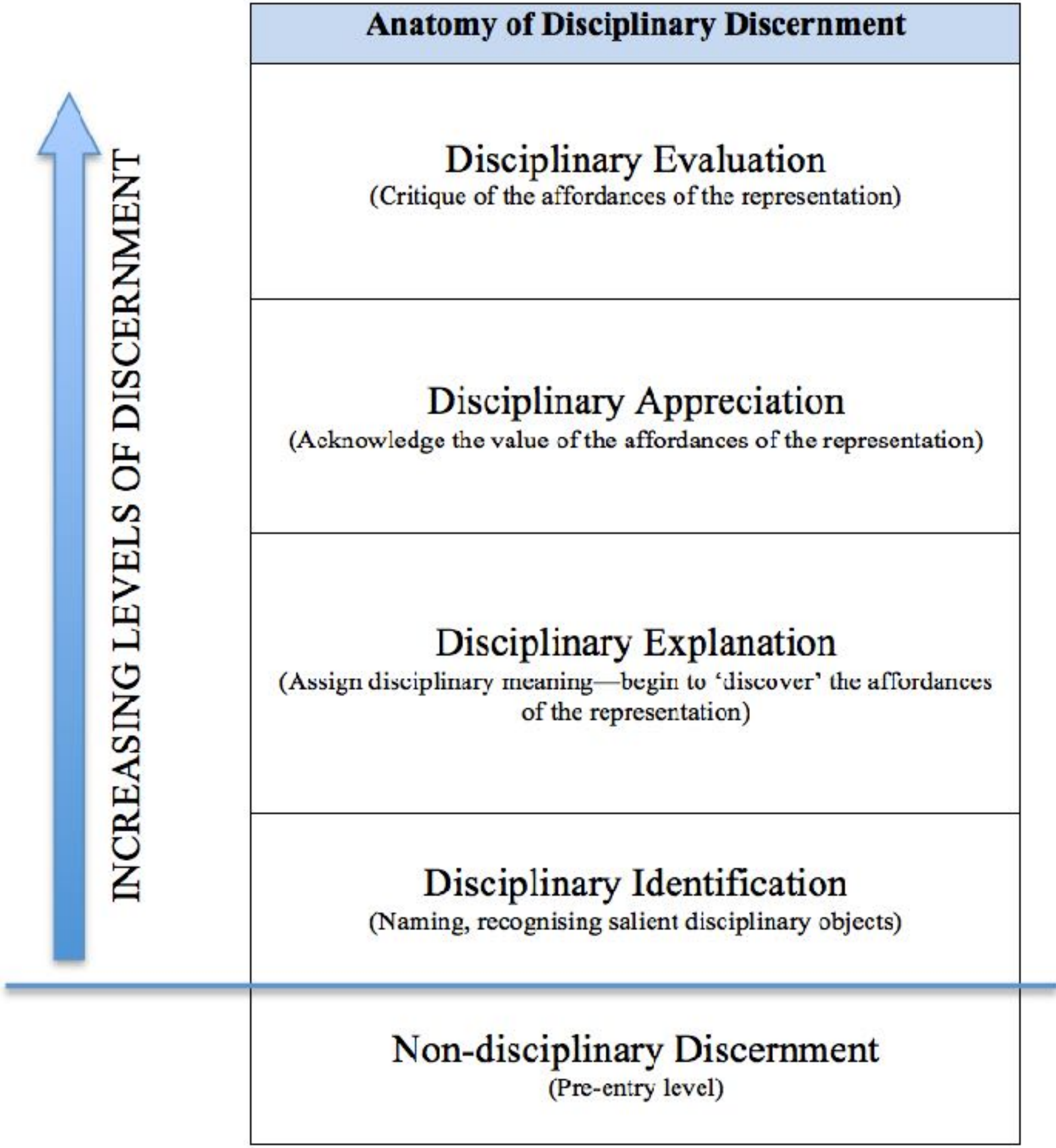

Disciplinary affordances in mathematics - Examples

- Functions $y = f(x)$
- Derivation $y' = f'(x) = \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
- Integration $I = \int f(x)\,dx$

Disciplinary discernment
-What would a student discern?

- Functions $y = f(x)$
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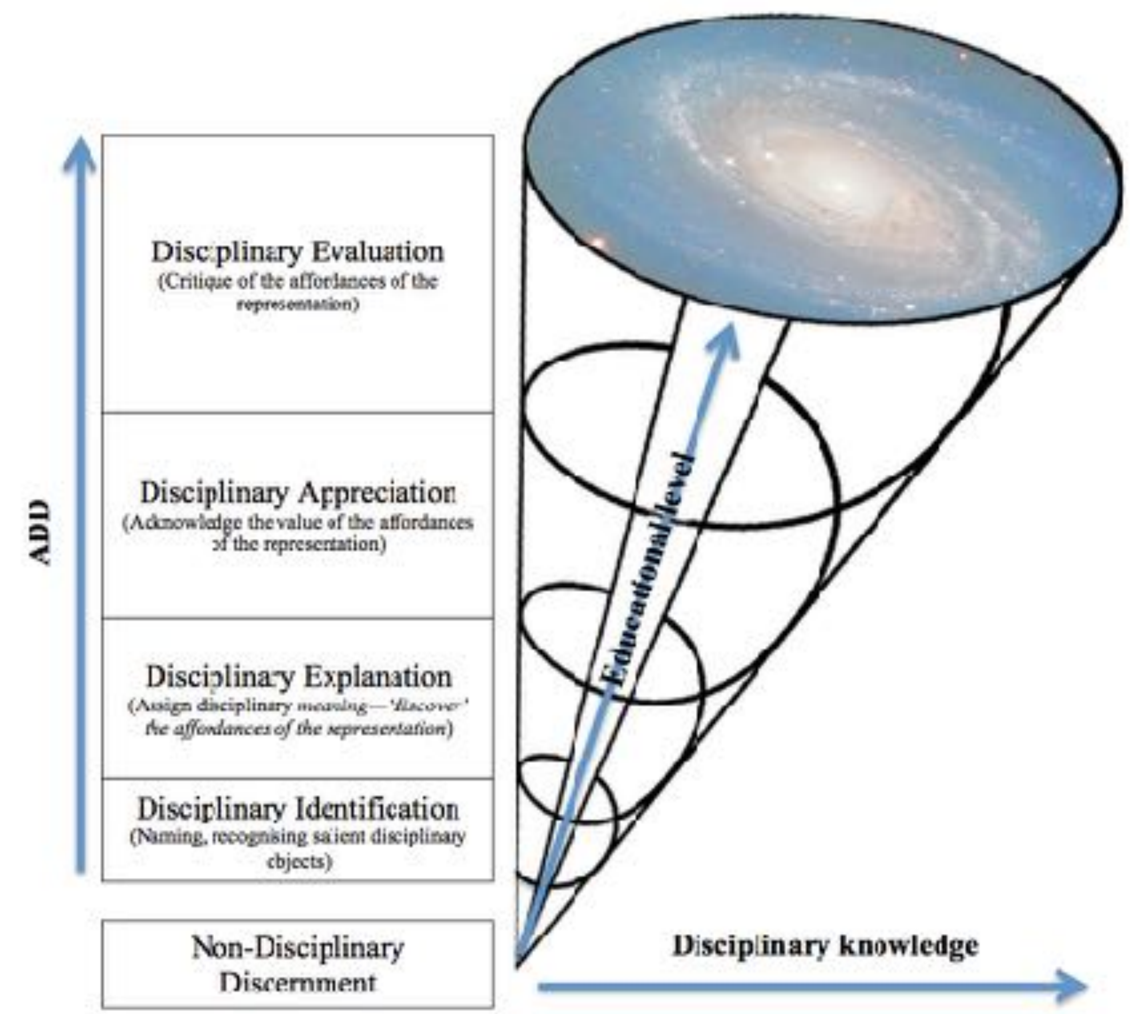
Towards optimizing teaching and learning

<u>**Knowledge construction**</u>
- a process involving **organizing** and **categorizing information** through **educational experience** that follows sequencing based on **the ADD** (Eriksson et al., 2014a,b).

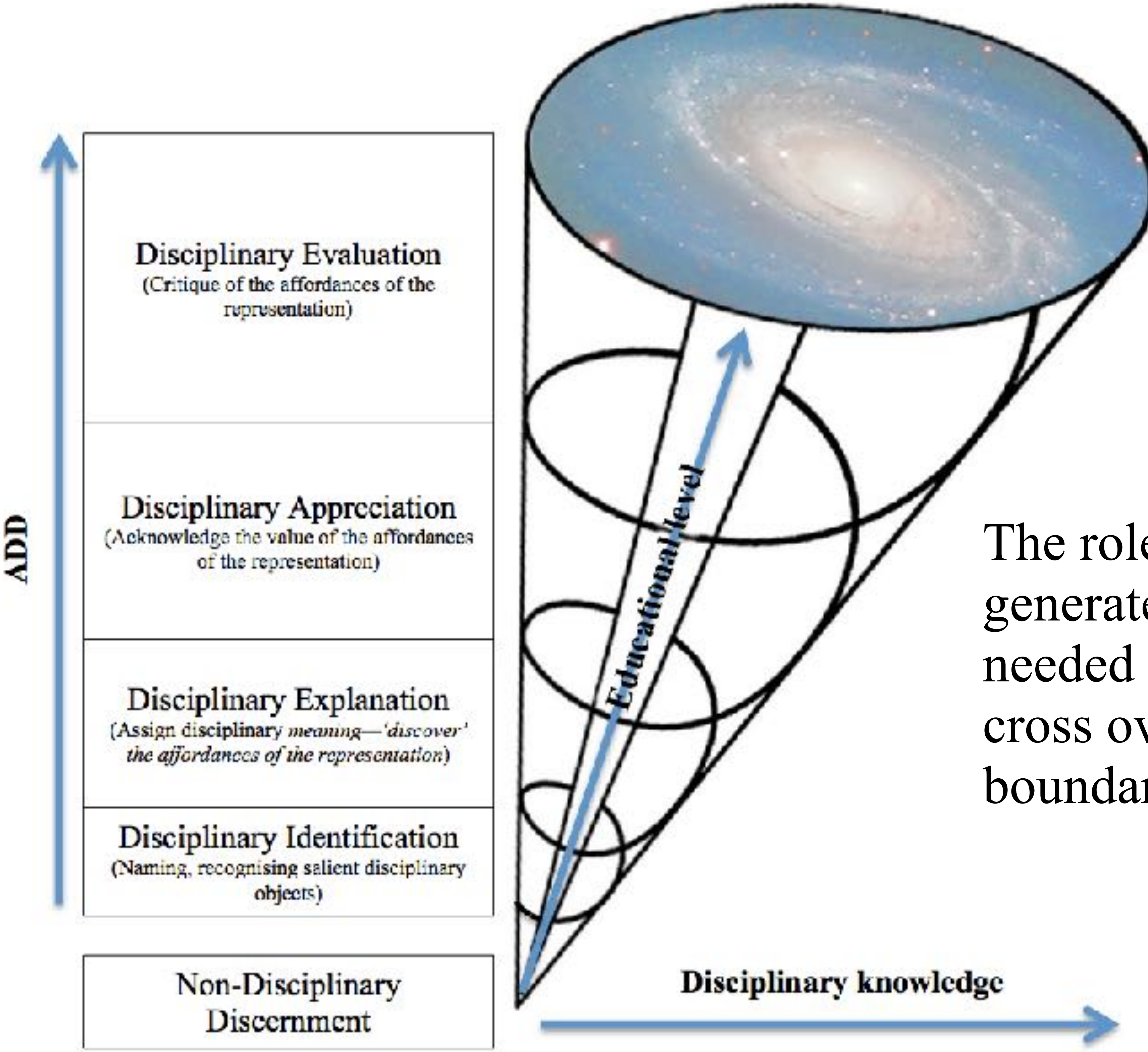


Growing into the discipline <-> Category boundary crossing

Not sufficient in itself and guidance is often required (see, for example, Mayer, 2004).

John Hattie (2012) highlights the roll of the teacher in this process.

The Spiral of Teaching and Learning



The role of the teacher is to generate the scaffolding needed to help the students cross over category boundaries in the ADD

So…

- The **ADD describes**, from an educational perspective, how **disciplinary knowledge** can be seen to **increase** as a function of a growing **competency to discern** disciplinary crucial **aspects** from a **vast array** of potential **disciplinary affordances** of a given representation, i.e **”reading” the representations.**

- **The Spiral of Teaching and Learning** describes the development from disciplinary "**outsider**" to disciplinary "**insider**".

Implications

- **Visible Learning central:** the **role of the teacher is crucial** for the success of the students in ***crossing category boundaries*** in the ADD (cf. Hattie, 2012).

- Teachers should **begin teaching sequences** with activities which **draw out students ideas** to find out **where they are** in the Spiral of Teaching and Learning (Exit tickets?).

- Then **teach them to discern relevant disciplinary affordances of representations at that level** (cf. Ausubel, Novak, & Hanesian, 1978).

Conclusions

The Spiral of Teaching and Learning

- A new framework for how to see teaching and learning mathematics using disciplinary-specific representations.

- **Teachers** should **focus on activities** that help students ***cross category boundaries*** (Northedge, 2002), without make the task too complicated (Ainsworth, 2008).

- Then **students** can be expected to **discern** and **discuss** details of representations used in a disciplinary discourse, i.e. **take control of their own learning and become their own teachers**.

Discussion

- What role does social semiotics take in

  A. mathematics education?

  B. mathematics education research?

- What is your experience concerning disciplinary discernment?

- How to approach disciplinary discernment in teaching and learning mathematic?

Thank you for listening!

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