PROSPECTIVE TEACHERS' CONCEPTIONS OF ANALYSIS AND BELIEFS ABOUT THEIR IMPENDING PROFESSION

Kristina Juter

Kristianstad University College, Sweden

Linnaeus University, Sweden

Pre-service teachers were enrolled in a study about their perceptions of mathematics concepts and beliefs about the mathematics teacher role. The students' perceptions and beliefs were studied through questionnaires, interviews and classroom observations. Three students were selected as cases in this paper where their links between the concepts limits, derivatives, integrals and continuity were classified in what resulted in an expanded set of prior categories (Juter, 2009). The students' beliefs about mathematics teacher identities, as defined by Holland, Skinner, Lachicotte Jr and Cain (1998), are discussed in connection to the nature of their concept images of the topics aforementioned. The students who emphasised mathematics in the teacher role were the students with weaker concept images.

BACKGROUND

Students studying to become mathematics teachers at upper secondary school go through university courses in mathematics as students but with, at least partial, focus on their impending careers as teachers working with the subject with new learners. Their identities as students, mathematics teachers and mathematicians develop from experiences of mathematics and situations of learning and teaching mathematics, including thoughts and feelings. These identities are then working as foundations for their behaviour (Holland & Lachicotte, Jr, 2007). The students' mathematical knowledge represented in concept images (Tall & Vinner, 1981) and its meaning in terms of importance, status and enjoyment impact their identities as mathematics teachers as well as mathematicians, which is in itself part of the teacher identity. Mathematics plays different roles to different teachers and their identities are characterized differently and to various degrees by their experiences of mathematics. In the present paper students' representations of mathematical concepts in analysis are analysed and compared to their views of teaching mathematics. The research questions addressed are:

- What beliefs do pre-service teachers have about mathematics and mathematics in their upcoming professions as mathematics teachers?
- How do these beliefs compare to the students' views of mathematical concepts in analysis?

The research questions are not disjoint, since beliefs about mathematics and mathematical knowledge partly develop from the same experiences of mathematical activity, and hence influence each other. Confidence from the own mathematical conceptions have been shown to lack correlation with actual mathematical abilities in some cases (Juter, 2005, 2006) and prior research points at the cognitive challenges analysis provides for students at university (e.g. Cornu (1991), Hähkiöniemi (2006), Juter (2006) and Viholainen (2006)). Research on relations between the nature of students' conceptual representations and the way they view the teacher role, their own and others, is useful in teacher education to help students become aware of their own abilities. Skott (2009) points to the danger of over interpreting the impact on practice of research of teachers' beliefs and knowledge since classroom situations are influenced by several social interactions, not only the ones controlled by the teacher. Teachers are however managers and advisors in the overall social interplay of their classrooms and possible effects on the pre-service teachers' practice are discussed from the point of view of the research questions posed.

THEORETICAL FRAME

Mathematics in teacher identity

Clusters of a person's self, created in various situations, form a person's identity (Holland, Skinner, Lachicotte Jr & Cain, 1998). They are interwoven and affect each other in the person's expression of identity in interaction with his or her surroundings (Gee, 2000-2001). Figure 1 shows an example of a simplified model of different identities affecting the mathematics teacher identity. Experience of teacher education has naturally a direct impact on a person's mathematics teacher identity, whereas a course in general mathematics may strengthen the person's mathematical identity and only later, in a teaching situation, influence the mathematics teacher identity. Less obvious situations may also affect the identity, in this case for example experiences from leading a team in soccer practice or being a parent. Focus of this paper is in the upper right part of the mathematics teacher section, i.e. on the mathematics role of the students' identities.

A mathematics teacher, or a student studying to become one, has many relevant kinds of beliefs that influence adaptation to the profession. Beliefs about themselves, mathematics, mathematics learning and teaching, and social settings are all important parts to consider (Leder, Pehkonen & Törner, 2002). Some beliefs can be isolated and others, at least partly, overlapping each other. Wilson and Cooney (2002) stressed the importance to take both pedagogical and conceptual beliefs into account in studies of mathematics teachers since one set of beliefs may evoke parts of the other. An effect of that is that the students own conceptions of mathematics affect their roles as mathematics teachers.

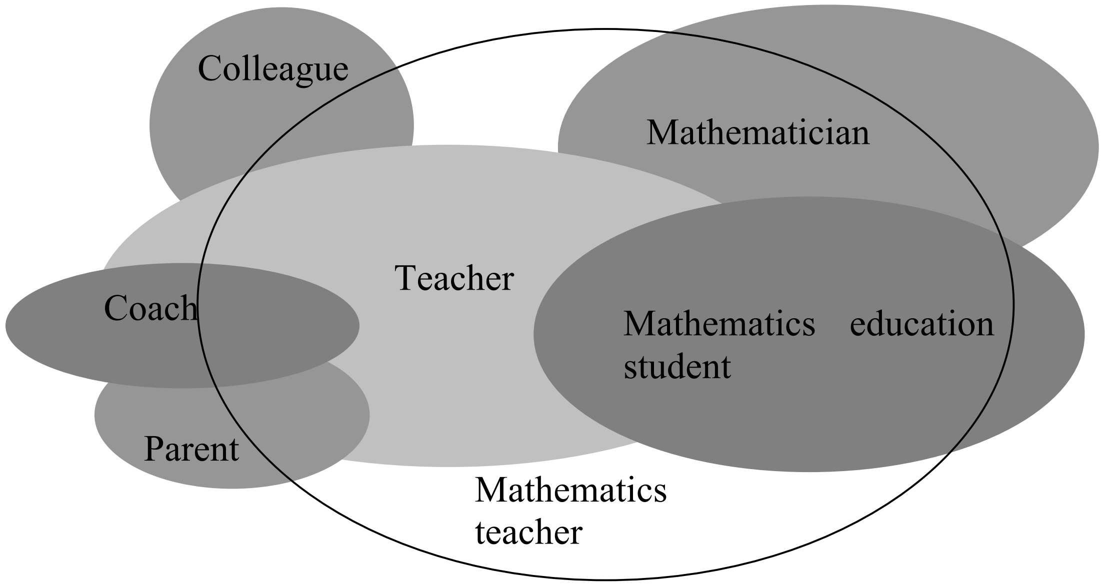

Figure 1. A model of examples of clusters of identities influencing the mathematics teacher identity.

New teachers or students doing their practice in classes are part of various *communities of practice* (Holland, Skinner, Lachicotte Jr & Cain, 1998) e.g. the learning situation of the class and the collegial context with the other teachers. The communities of practice are framed by *the figured world* of the learning environment, i.e. the cultural processes created in the societies of being a mathematics teacher or a colleague. The figured world consists of the traditions and actions characteristic for the learning environment. Identities are formed by participation in communities of practice rendering them dynamically dependent on beliefs, attitudes and experience. Liljedahl, Oesterle and Bernèche (2009) categorized literature on research about beliefs and concluded that beliefs are continually changing and they found no evidence that systems of beliefs are stable. A personal identity is however more stable than beliefs as the identity determines who the person is as opposed to how the person responds to a phenomenon.

Categories of conceptual representations

Tall's three worlds (2004, 2008) describe mathematical development in three different modes, *the conceptual-embodied world* with an emphasis on exploring activities, *the proceptual-symbolic world* focusing concepts' dual features as objects and processes expressed in symbols or *procepts* (Gray & Tall, 1994), and *the formal world* where mathematical properties are deduced from the formal language of mathematics in definitions and theorems. An individual's concept image (Tall & Vinner, 1981) is developed through the three worlds in various trajectories allowing him or her to understand concepts differently. Skemp (1976) distinguished understanding a concept from its core features, *relational understanding*, which enables implementation of the new concept to existing concept images (which is how Hiebert and Lefevre (1986) defined understanding), from understanding by just being able to perform a particular operation in what he denoted *instrumental understanding*. Pinto and Tall (2001) described two ways of learning new concepts. A *formal learner* uses definitions and symbols as a ground in the axiomatic-formal world (Tall, 2008), whereas a *natural learner* logically deduce new concepts from working with his or her concept images in the conceptual-embodied world and the proceptual-symbolic world (Tall, 2008). Both formal and natural learners can have relational understanding, if concepts are successfully integrated in their concept images. An attempt at learning, either way, may however result in instrumental understanding. I have, from the definitions of natural learner, formal learner, Skemp's definitions of understanding and Tall's three worlds, created a set of definitions (presented in table 1) to categorise students' links between concepts. The classification is a development of an earlier set of categories (Juter, 2009).

Table 1. Definitions of links between concepts in the classification.

Type of link	Definition
Valid link, procedural (vp)	True relevant link with focus on calculations or applications
Valid link, naturally conceptual (vn)	True relevant link revealing a core feature of the concept, not formal
Valid link, formally conceptual (vf)	True relevant link formally revealing a core feature of the concept
Irrelevant link, no reason (ir)	No actual motivation for the link is provided
Irrelevant link, no substance (is)	Peripheral true link without substance relevant for the concept
Invalid link, misconception (im)	Untrue link due to a misconception of the concept
Invalid link, counter perception (ic)	Untrue statement contradicting prior statements of the student

OUTLINE OF THE STUDY

Students from four groups, two from each of two different universities in Sweden, were part of the study. All students, a total of 42, were pre-service teachers in mathematics who were studying to teach grades 7 to 9 and upper secondary school. The study started with one group, group 1, two years before the remaining three groups were added to the project as table 2 indicates.

Table 2: Data collection times for the groups in years 1 to 5

Group	Autumn 1	Autumn 2	Autumn 3	Spring 4	Autumn 4-5
1	Questionnaire, Tasks	Interview 1	Interview 2	Interview 3 observation	
2-4			Questionnaire, Interview 1	Interview 2	Interview 3 observation

The first data collection in three of the groups (1, 2 and 4) was at the beginning of the students' analysis course where they filled out a questionnaire aimed at revealing their pre-knowledge of the concepts investigated. The questions were openly formulated for the students to be able to answer without influence from other formulations. The questionnaires were used to determine which students to ask for further participation in the interviews. Their different ways to respond were represented in the interview sample of students. The purpose of the questionnaire in the fourth group (group 3) was the same but it was somewhat differently designed since the students had completed their analysis course at the time of data collection. All students in the other groups were examined in the beginning, and after or at the end of their analysis course. The post examinations differ according to circumstances the different years. The post examination in group 1 consisted of the course exams and tasks, as part of the course, designed to test for example the students' understanding of the limit definition. In groups 2 and 4, the post examinations were only done among the students participating in the interviews since the others were not further investigated. All students in all groups filled out questionnaires (and tasks and exams in the first group). All interviews were individual and audio recorded.

The aim with the first interview in groups 1 and 3 was to investigate the mathematics links between the concepts. The questions and tasks were quite open at first to let the students chose their own formulations of the concepts. Then the instruments used were more directed to different aspects of the concepts. One instrument was a table of words and phrases used with the purpose to work as stimuli for the students to recall parts of their concept images. The students were to describe the evoked parts or say if there was no recollection linked to a particular concept. This matrix was used in all groups about a half year to a year after their analysis course since conceptions alter as time goes by. Four graphs with different characteristics linked to the four concepts studied were also given for the students to determine whether or not they have limits, are differentiable, integrable and continuous. The students also got fictitious student solutions to discuss from a teacher and a mathematics perspective. The first interview in groups 2 and 4 did not include the matrix of recalled links since their analysis courses were the same semester. The matrix was presented to those students in the second interview about six months after the analysis courses. The open questions used in the questionnaire were used again in the interviews to reveal how the concepts had developed in the students' concept images. The students' coming profession as mathematics teachers was also explored.

The second interview focused more on the students' roles as mathematics teachers. The students were particularly asked to describe themselves as mathematics teachers, what characterize a good or bad teacher from their own experiences and to select important teacher features from a given list. The different types of questions were designed to bring out the students' views of mathematics teachers' roles in different settings. The variation enabled a triangulation of what they selected to be the most important features, e.g. mathematics subject matter or social interaction with students, to use for descriptions of teachers. The conceptual issues were addressed again in various forms. The first and second interview comprised the same components together for all students, but differently disposed depending on the time of their analysis courses.

The third interview was linked to the students' work in classes where observations with short prior and post interviews were done. The purpose was to see if the students' actions in the classrooms were coherent with their narratives from the interviews. The observations were recorded either with a video camera, audio recorder and field notes, or just field notes. Four students were, to this date, unable to participate in this part of the data collection since they did not have access to a class. Since a class is required for the observations, there have been some delay due to for example working situations, but as soon as there are opportunities for observations, they are done. The times of the observations in table 2 are hence flexible as the data collection is still in progress. Simulated written situations were discussed as a complement in some of the cases as a precaution when the students did not seem to get access to a class.

RESULTS

The ten interviewed students' belief about the mathematics teacher's role is depicted in table 3 along with their views about mathematics. The students were divided into three groups depending on the character of their links (as defined in table 1) between the concepts from the matrix described in the outline of the study. One student from each group was selected in this paper (table 4) as examples of how the students' conceptual links relate to their views of the mathematics teacher profession. The selection was therefore also based on the results of the students' views presented in table 3. The 3 students in the first group, group A, had few valid links of which none were formally conceptual (vf). Several links were irrelevant (ir, is) or invalid (im, ic). The 4 students in group B, had more valid links than the students in group A, but also no formally conceptual ones. The students had few irrelevant or invalid links. Group C comprised 3 students with many valid links, including formally conceptual links and very few irrelevant or invalid ones. The letters by the students' names in tables 3 and 4 indicate which group they belong to.

Table 3. Students' views of the teacher's role and mathematics

Teacher's role		Students	Beliefs about mathematics	
Mathematical content before social interaction	Teacher as a friend or peer to the students	Alex, A	Language and relations	Difficult but fun
		Celia, A	Numbers and symbols	
		Linus, B	Logic	Fun if not difficult
Mathematical content before social interaction	Teacher as a leader or guide to the students	Felix, C	Relations, problem solving not routine calculations	Easy and fun
Social interaction before mathematical content	Teacher as a leader or guide to the students	Simon, C	Problem solving not routine calculations	Difficult but fun
		Ian, A	Descriptions of reality	Difficult but mostly fun
		Kitty, B	Logic and rules	Fun
		Paul, B	Numbers and applications	Easy
		Robin, C	Language and logic	Easy and fun
		Jessica, B	Numbers, symbols and calculations	

The students who thought that the teacher should be a friend rather than a leader also stressed the importance of mathematical content before social abilities with the students, such as communication, and vice versa. None of the students emphasising mathematical content thought that mathematics is easy.

Table 4. Students' links between concepts categorised by the definitions in table 1. The students are arranged by their links in groups A-C as indicated after their names.

Student	Limit	Limit	Derivative	Derivative	Integral	Integral	Continuity	Continuity
Celia, group A	vp 0 vn 1 vf 0	ir 2 is 2 im 3 ic 0	vp 1 vn 2 vf 0	ir 1 is 2 im 1 ic 0	vp 1 vn 0 vf 0	ir 0 is 0 im 1 ic 0	vp 0 vn 1 vf 0	ir 0 is 0 im 3 ic 0
Kitty, group B	vp 2 vn 0 vf 0	ir 0 is 0 im 1 ic 0	vp 2 vn 5 vf 0	ir 3 is 0 im 0 ic 0	vp 4 vn 3 vf 0	ir 1 is 2 im 0 ic 0	vp 0 vn 1 vf 0	ir 0 is 2 im 1 ic 0
Simon, group C	vp 5 vn 5 vf 3	ir 2 is 1 im 0 ic 0	vp 6 vn 1 vf 6	ir 1 is 1 im 0 ic 0	vp 7 vn 5 vf 1	ir 0 is 1 im 0 ic 0	vp 3 vn 1 vf 0	ir 3 is 1 im 0 ic 0

Celia from group A had few links in table 4, only 21, as she was struggling with the analysis course. 6 links were valid, 7 irrelevant and 8 were invalid showing a weak understanding of the concepts investigated which also became apparent from her responses to other tasks in the data collection. Celia was aware of the fact that her mathematical skills were weak, but she saw that as one of her strengths as a mathematics teacher to be able to understand her struggling students' situation. As a teacher she wanted to show her students applications and the joy of mathematics. Celia wanted to be a friend with authority to her students and claimed not to be nervous before her lessons. She thought mathematical skills were more important for a mathematics teacher than interaction with the students, but very little mathematics was done at her lesson as she went around in the classroom talking to the students about their interests, occasionally trying to stop the students from playing around.

Kitty, in group B, had 27 links in table 4 in all of which 17 were valid. Kitty also had rather few links compared to the other nine students. She showed a high level of natural conceptual understanding of the concepts derivative and integral. 8 of Kitty's links were irrelevant and 2 were invalid. She was studying to become a teacher in mathematics and science. She thought that science lessons are harder to prepare for than mathematics lessons because in mathematics she could let the students work in their textbooks without preparation, which is not possible in science lessons. She felt nervous if she was uncertain of any part of what she was going to teach. Kitty thought that the relation to the students is more important than the mathematics skills of the teacher, but she emphasised that mathematical skills are important for teachers to be able to explain in a varied way. She thought applications are important as means to justify the mathematics taught. She described her relation to the students as a leader. She said she wanted to be a friend to her students, but that it is not possible from a professional point of view. She was leading her class from the whiteboard inviting the students to answer questions acting as she described in the interview.

Simon from group C had 53 links in table 4. He had 10 formally conceptual links of his total of 43 valid links, leaving 10 irrelevant links. Simon showed an overall strong and rich concept image through the data collection. He stated that he did not like proofs, yet he was the one with the highest rate of formal links (vf) among all ten students. Simon taught mathematics and science. He was calm and started his lesson at the whiteboard engaging his students to be part of solutions and reasoning. The entire lesson was focused on mathematics while Simon praised and helped his students, about half the time at the whiteboard and the rest while the students were working in their textbooks. He wanted to have even more lectures at the whiteboard in the future to be able to control what the students encounter in mathematics. He saw mathematics as problem solving, not calculations, and the use of applications as an important method to inspire the students and to make them understand how mathematics can be practical.

CONCLUDING REMARKS

Simon lifted social skills before mathematical skills as important for a teacher in the interview. His actions in the classroom showed a strong focus on the mathematics with explanations to all questions and details, a combination of mathematical and classroom social skills. He acted as a leader like he had stated a teacher should. All three students in group C with strong valid concept images regarded teachers as leaders rather than peers to their students. Celia put mathematical content as more important than social abilities with the students, but her lesson was all about social interaction, about other things than mathematics, with the students. Almost no mathematical activity occurred during the lesson. Of the three students regarding a teacher as a friend to his or her students, all were prioritizing mathematical content before student relational skills and two of them were from group A with the weakest concept images. A weak concept image in a community of practice (Holland, Skinner, Lachicotte Jr & Cain, 1998) of a classroom may force the teacher to compensate for the lack of mathematical competence and, like Celia did, focus on processes requiring other skills. Celia was behaving contradictory to her beliefs about how the practice ought to be, i.e. with an emphasis on mathematics. Not all students with weak concept images were aware of their invalid or irrelevant links. Alex, from group A, had a large number of links of which a majority was invalid or irrelevant. The numerous links gave him a false feeling of competence which may lead him to teach inaccurately.

Further analysis of the students' teacher role is in progress. Their concept images will also be described in more detail from all parts of the data collection to answer questions about their mathematical identities. Issues of presentation of results from the broad research aim need further elaboration.

References

Cornu, B. (1991). Limits. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 153-166). Dordrecht: Kluwer Academic Publishers.

Gee, J. P. (2000-2110). Identity as an analytic lens for research in education. *Review of Research in Education* 25, 99-125.

Gray, E., & Tall, D. (1994). Duality, ambiguity and flexibility: A procedural view of simple arithmetic. *The Journal for Research in Mathematics Education* 26(2), 115-141.

Hiebert, J & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics.* London: Lawrence Erlbaum Associates publishers.

Holland, D. & Lachicotte, Jr, W. (2007). *Vygotsky, Mead, and new sociocultural studies of identity.* Cambridge: Camebridge University Press.

Holland, D., Skinner, D., Lachicotte Jr. & Cain, C. (1998). *Identity and agency in cultural worlds*. Cambridge: Harvard University Press.

Hähkiöniemi, M. (2006). *The role of representations in learning the derivative.* Doctoral thesis. University of Jyväskylä, Finland.

Juter, K. (2005). Students' attitudes to mathematics and performance in limits of functions. *Mathematics Education Research Journal* 17(2), 91-110.

Juter, K. (2006). Limits of functions as they developed through time and as students learn them today, *Mathematical Thinking and Learning* 8(4), 407-431.

Juter, K. (2009). Development of students' concept images in analysis. *Nordic Studies in Mathematics Education* 14(4), 65-87.

Leder, G., Pehkonen, E. & Törner, G. (Eds.), (2002). *Beliefs: A hidden variable in mathematics education?* Dordrecht: Kluwer Academic Publishers.

Liljedahl, P., Oesterle, S. & Bernèche, C. (2009). Beliefs as dynamic: old light through a new window. Paper presented at *MAVI 15* in Genoa Italy, 2009.

Pinto, M. & Tall, D. (2001). Following student's development in a traditional university classroom. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th conference of the International Group for the Psychology of Mathematics Education* (pp. 57–64). Utrecht: The Netherlands.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching* 77, 20–26.

Skott, J. (2009). Researching teachers' beliefs and knowledge: What (not) to expect in terms of impact? In C. Winsløw (Ed.), *Nordic Research in Mathematics Education* (pp. 99-108). Rotterdam: Sense Publishers.

Tall, D. (2004). Thinking through three worlds of mathematics. In M. Johnsen Høines & A. B. Fugelstad (Eds.), *Proceedings of the 28:th conference of the International Group for the Psychology of Mathematics Education* (pp. 281–288). Norway: Bergen University College.

Tall, D. (2008). The transition to formal thinking in mathematics. *Mathematics Education Research Journal* 20(2), 5–24.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics* 12, 151-169.

Viholainen, A. (2006). Why is a discontinuous function differentiable? In J. Novotna, H. Moraová, M. Krátká & N. Stehlíková (Eds.), *Proceedings of the 30'th conference of the international group for the psychology of mathematics education,* (pp. 329-336). Prague: Charles University.

Wilson, M. & Cooney, T, (2002). Mathematics teacher change and development, the role of beliefs. In C. Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 127-147). Dordrecht: Kluwer academic publishers.