Teachers' awareness of student learning

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Abstract

We report from a project aiming at stimulating primary and secondary teachers to work with open-ended activities, and using accounts-of to develop their awareness of the results of these activities. Results show that accounts-of seem to have a good potential of developing teachers' awareness of student reasoning and thinking, i.e. giving them a better stance for following the learning of the students, and that open-ended tasks seem to have a great potential for a better inclusive mathematics teaching by not demanding special tasks or special instructions for certain students or smaller groups of students.

Background

In this paper we report from a project where we during the school-years 2006/2007 and 2007/2008 worked together with teachers in a small municipality in the Southern part of Sweden in order to stimulate the development of their mathematics teaching. The project was headed by the author and planned and performed in cooperation with Annika Palmgren, Svedala commune, and Pesach Laksman, Birgitta Lansheim, Ulla Öberg, all in different ways connected to Malmö University College. It was funded by a grant from Sparbanksstiftelsen Skåne.

Traditionally mathematics education at least in Sweden has a heavy focus on developing and exercising skills. And teaching is often based on some kind of prototypical tasks, which the teacher (and the book) demonstrates how to interpret and solve. Thereafter the students are allowed to practice on identical or very similar types of tasks, and the aim is that this kind of problem will become a kind of routine problem that they, when confronted with later on, immediately will know how to solve. In this tradition there is naturally a strong focus on right or wrong. For students with special needs special care is taken to make the tasks manageable, often resulting in a reduced mathematical content and an even heavier emphasis on skills, in the way that they are separated into exercising different more elementary and reduced skills.

The implicit learning from this tradition has been analysed for example by Lampert (1990). She notes that working with mathematics in this tradition gives the student the impression that the important thing is to learn to follow the rules that the teacher or the book gives. If you do not quite understand them, at least learn how to use them. To know mathematics becomes equivalent to remembering and using the appropriate rule, and right or wrong is determined by authority, either by the teacher or by the list of answers at the end of the book. As a consequence the best way of learning mathematics is to listen carefully to the teacher's expositions and to practice often. Different aids like a list of formulas can be an important or necessary tool for solving different problems. Overall this implicit learning and other kinds of experiences from this kind of traditional mathematics teaching often make students feel that mathematics is not for them, or sometimes when the demands on gifted students are too low they find it boring and uninteresting.

In many reform programs all over the world different ways of trying to change this tradition are proposed (see e.g. Kilpatrick e.a., 2001; Lampert1990; Sullivan e.a., 2006). These programs often stress an increased work with genuine problem solving leading to overt reasoning and discussions in the classrooms, with a focus on developing the student's mathematical thinking. Some researchers as Sullivan e.a. (2004) and Sullivan e.a. (2006) have explored the potential of open-ended tasks for managing a more inclusive kind of mathematics, building on what students already know, and creating activities from which all students can benefit, despite their different knowledge.

The need of developing practical and functional models for how to manage such an enterprise, creates a demand for in-service training. However the traditional in-service training (at least in Sweden) often consists of some "expert" teacher showing other teachers how to do. Their messages can be very normative with very fixed opinions on what works and equally fixed opinions on what type of activities are bad. Often the reasons for these opinions have a poor scientific base.

In the project we want to report from, our aim was to stimulate teachers to work with open-ended activities that give students opportunities to develop their problem-solving behaviour and thinking. We also wanted to stimulate the teachers to develop their awareness of their own actions and of the results of the activities they presented the students with, in the form of student behaviour and ways of reasoning. Instead of telling teachers how to do, we wanted to empower them by stimulating their own reasoning and thinking, starting in their own beliefs and experiences.

Theoretical background

The overall framework in our project, for teacher development and for the research pursuing the kinds of processes this entails, have mainly been adopted from the Discipline of Noticing as formulated in Mason (2002). This means that teacher learning is seen as consisting of becoming more aware of distinctions and relations between different phenomena and behaviour, and of an increased ability to see increased opportunities in a situation. In such a research project data therefore need to be complex enough to mirror what is genuine in a situation, and that the process of analysing is careful enough to keep this genuineness.

In order to develop the awareness of the teachers for what is happening in the classroom, we have therefore adopted a tool presented in Mason (2002) which is called making accounts-of. In an account-of you have a focus on describing what has happened in contrast to an account-for where you in addition to making a description, also give interpretations of and explanations for the reasons why the event developed as it did, as interpreted by the person making the account. The starting point for an account-of is something that happens in the classroom that you notice spontaneously. Often this entails elements of surprise or something that you do not quite understand. This can be about your own feelings or reactions or your students' behaviours and reactions. The task then is to account of this event as descriptively as possible, i.e. leaving interpretations beside. But in order to do that you have to find a way to remember what it was that you noticed, and what your emotions and spontaneous reflections was all about. Since you often are in the middle of teaching when the situation occurs, this is not always easy. One way to manage this is to just write down a few key words that later will have the possibility of acting as an entrance to your memory of the event. Then, perhaps a few hours or days later when you can find the time, it will be possible for you to write down more about what happened and what it was that caught your attention. You can even try to label the account-of, but the important thing is that your account is as descriptive as possible. The reason is twofold. Firstly, by focusing on the descriptive side of the event you can become more aware of what is happening in contrast to your own interpretation of what is happening. Secondly, when you share a description with colleagues, they will spontaneously interpret what is happening from the background of their own experience, in a way that they will not do when listening to an account-for. In this way accounts-of can give a base of identifiable events, which when they are shared in a group of mathematics teachers generate different interpretations based on the participants different experiences, and thus can be used as a basis for exchange of these experiences.

Our work with teachers is also inspired by the Cognitively-Guided-Instruction-project (CGI) in the USA (Fennema e.a., 1996; Franke e.a. 2001). This project had a focus on stimulating teachers to use problem solving in mathematics and to focus on the development of student thinking as one of the most important goals for mathematics education. The role of the researchers in the project was as experts on problem solving and the strategies students use to solve different kinds of problems. The teachers on the other hand, and not the researchers, were seen as the professionals for developing activities in the classroom (Fennema, 2002). As a result of the project they also developed a scheme

for analysing teacher's engagement with student thinking (Franke e.a., 2001), that in some modified form we will use to analyse the development of the teachers in our project. This engagement with student thinking using materials from their own classes that the participants have the possibility to report in a group of other teachers, where they can be discussed and reflected upon in a professional community, can be seen as a hallmark of successful in-service teacher programmes (Kazemi & Franke, 2004).

In our project we have tried to stimulate the participants to use open-ended tasks and investigative activities of different kinds. One source of inspiration for this work has come from the work with open-ended questions by Peter Sullivan and colleagues in Australia (Sullivan e.a., 2004 ; Sullivan e.a., 2006) working to find out productive ways of managing open-ended tasks in an inclusive classroom. Open-ended tasks are here seen to start within a context that has the capacity to engage all students, and that has more than one possible solution. This openness gives the students a possibility to spot and investigate patterns in the different solutions, which also can encourage the students to work in a more systematic manner finding all the solutions. This investigative work very often also leads into trying to find new questions worth investigating in their own right. We have also been inspired by the work aiming at exploring the potentials of different kinds of investigative tasks by John Mason in the UK (Mason & Johnston-Wilder, 2006).

Research Questions

- In what ways can work with accounts-of stimulate teacher's awareness of and engagement with student thinking and learning?
- What potential have Open-ended tasks for facilitating a genuine inclusive mathematics education?

Method

At the outset of the project we contacted the educational department of a municipality in the southern part of Sweden, and asked them if they were interested to support a project where mathematics teachers at different levels of the school system would get the opportunity to develop their mathematics teaching. We asked for participants in three groups of teachers that have similar planning and teaching conditions: A group of pre-school teachers (Pre) that have very limited time for planning and not especially specific curricular goals working with children from 0 to 6 years old. A group of primary teachers (Prim) with responsibility for almost all of the subjects taught in a class working with students from 7 to 12 years old. And finally a group of secondary teachers (Sec) with responsibility for about two subjects including mathematics, and working with students that were 13 to 16 years old. We also asked for participants that were interested in and willing to develop their ways of working with mathematics. The selection process resulted in 10 participants in the Pre-group, 10 in the Prim-group and 6 in the Sec-group, and the project lasted during the school-years of 2006/2007 and 2007/2008.

The primary activity we offered were meetings of a workshop character in these basic groups. These were held regularly every third week, and consisted of two different kinds of activities. At each of the meetings one or more often two members of the research group were present. Usually we started by letting each participant report one or two of the accounts-of that they had gathered since the last meeting, followed by discussions and exchanges of experiences. Thereafter we introduced different kinds of activities that aimed at stimulating the participants to try out some open-ended tasks in their classes until the next meeting. This could be by letting them try out different examples of open-ended tasks or by giving them some report of research findings that we considered relevant, depending on the themes that the discussions had opened up. When this is said it is important to remember that all the time it was the teacher himself/herself, that were responsible for planning and selecting the kind of activities and tasks to use in his/her teaching depending on the topics covered in the class and the age of the students.

A second activity we offered were workshops twice a semester, where all the participants from the different groups met and worked in mixed groups. The rationale for these meetings was to stimulate

discussions that were crossing the borders of the usual groups and working conditions, and to stimulate a more longitudinal perspective of the mathematical development of different students.

Since our primary goal was to study the impact of the meetings for the development of the teachers, the meetings in the basic groups were all audio-taped, and written versions of the accounts-of were gathered continually. Meetings in the big group were not recorded. Due to limits in time and resources we did not perform any observations of lessons. At the outset and the end of the project the participants filled in a questionnaire about their view of themselves as a mathematics teacher. At the end of the first school-year they were given an evaluative task and were interviewed, and at the second school-year they were given a free evaluation task that were formulated as

> *Tell us about your experiences of your own developmental process during the project. Write at length and with as many distinctions as possible.*

In this paper we will almost exclusively report from this last evaluative task, and due to the theme of the conference only from the Prim and Sec groups.

Results

So far only the evaluations have been analysed in any detail. So the main results reported will be from the written evaluations. The analyses have so far resulted in an identification of themes that keep recurring in the evaluations of the different teachers, and the results will therefore follow these themes.

Accounts-of

Writing accounts-of have been to most of the participants a fruitful tool for documenting student learning. Some have even adopted them for other school subjects as well. The participants also witness of an increased awareness of the students' different ways of reasoning and that they tend to have more prolonged discussions of a fewer number of tasks then before. However the accounts-of have also been a somewhat demanding tool that have needed time to get accustomed to. Not everyone find it easy to write down what they notice. It takes time. Time you must find worthwhile.

Most of the teachers think that the accounts-of is a good tool for increasing the awareness of nuances in student's learning, and for making new things become visible in different situations. In this way the accounts-of function as a mean to broaden the perspectives of the teacher. They also work as a tool to document the learning of individual students, and thus becomes a valuable means for use in contacts with colleges and parents. The participants also mention that the accounts-of has helped them to better understand the problems students can be in concerning the understanding of mathematics. This is witnessed in the following citation, (where the respondent teacher is given in parenthesis by group and number):

> *Gradually … more easy and natural to notice what happened within the students (*Prim 4*)*

Although the accounts-of are beneficial many participants also witness about the difficulties they have had in adopting them into a regular habit. Some say that they have been hard to become accustomed to and to carry through, especially at the beginning of the project.

> *They must be done at once, something I have had difficulty to manage. (*Sec 4*)*

> *The part that has been hardest during the project … but … I was forced to look at and listen more to the students … saw and heard my students developing (*Prim 7*)*

There was also a difference in the number of accounts that was produced in the different groups. In the Pre- and the Prim-groups almost all of the participants wrote accounts-of at a regular pace, and some of these were quite elaborate. But in the Sec-group some of the participants had very hard to overcome the threshold of starting to write down an account-of. These teachers thought that it is much easier to tell about an account-of instead of writing it down, remarking that in this way you get more nuances

that are easily lost in the process of writing. This resistance to writing was also explained as a preference of interest or cognitive style, as a mathematics teacher in the secondary school you are more likely not to be good at writing or especially interested in it.

Open-ended tasks

The majority of the participants report that the open-ended questions have given all the students challenges, more so than by using traditional tasks. Students have also enjoyed these tasks more and been looking forward to the mathematics lessons. They have also listened more actively to each other during the discussions and developed their confidence in working with mathematics.

Open-ended tasks were especially tried out in the Prim-group, where one teacher at the beginning of the project in her year 4 class even contrasted a traditional closed task with a reformulated open variant in order to find out what the students thought about the different types of tasks. The closed task she had chosen was task 1:

Emma makes 80 SEK per hour. How much has she made in all, if she works for 5 hours?

and the open variant was formulated as task 2:

Emma made 400 SEK. How many hours did she work, and how much did she make per hour?

Table 1A. Student notes in response to task 2: Group A.

80 kr	5 timmar
100 kr	4 timmar
25 kr	16 timmar
12,50	32 timmar
6,25	64 timmar
≈3,20	128 timmar

Table 1B. Student notes in response to task 2: Group B

15 min	400*4=1600
7½ min	8=3200
3¾ min	6=6400
1 7/8 min	32=12800
0≈95/100	64=25600
0≈42½/100	128=51200
0≈21¼/100	256=102400

Then she asked her students:

Which task did you enjoy the most?

And to her surprise one of her students answered:

I liked the open task better, because in the first one you don't learn anything, you only show if you already know how to solve it.

The majority of her students agreed that this was the case. As an example of what the group work resulted in, she gave the pieces of results displayed in table 1A and 1B. They have been typed from the original scrap notes as truly as possible preserving the original Swedish language.

In table 1A the students start with the data of task 1, but then they give the most obvious solution, i.e. 4 hours and 100 SEK per hour. After that they start to repeatedly halve the money earned per hour, resulting in very long work hours. Group B on the other hand, as can be seen from table 1B, starts by halving the time spent on the work resulting in "fast cash". We can also note the abbreviated notation for the multiplications in the right column resulting in a free use of the equal sign. Interesting to know

is that the students in this class had not previously met other fractions than halves and fourths, but as can be seen from table 1B we get information about their ability to handle halving fractions in a more general way. The notation 95/100 can be understood if you make 1 min into hundredths. Then one whole and seven eights will become approximately 188 hundredths which will be about 95 hundredths when halved. So retaining the unit of one minute the fractions given are surprisingly correct. The only odd thing is the use of zero as perhaps more denoting a small number than giving a more correct approximation of the fractions given.

To most of the students the open-ended tasks made mathematics more enjoyable and engaging, but not to everyone at least in the first year. Those students whose self-image as high-achieving rested on their speed of completing tasks in the book, open tasks made a threat. They took to long a time and it was very frustrating not being able to know whether you had got the right answer or not. To other students the diminished focus on right or wrong was relieving, and many struggling students made apparent improvements as can be seen in the following quotations:

> *The existence of more than one solution makes it even possible for a "slow starter" to come up with ideas, and it is not as easy to remain passive as when there only is one answer. (*Prim 2*)*

> *The "weaker" students … suddenly nearly all of these students began to develop. They are coming closer to the others in giant leaps.* (Prim 7)

This brings us to the topic of inclusion.

Inclusion

Some of the teachers in the Prim-group witness about how the open-ended tasks helped them to manage a more inclusive teaching as in the following example:

> *I have worked much with open-ended questions and suddenly the problem with age-integration and integration of special school students disappeared. There was no need for special instructions for only a small part of the class. … Now the children are more active, and can take part in investigations and make discoveries on their own level. It feels as if now the children seek knowledge instead of expecting me to deliver it by instruction. … They have no hurry and do not compete about leading the race, since their is no race.* (Prim 5, year 2-3)

Also in the Sec-group some teacher commented on the topic of inclusion:

> *To have within the same class students that are very capable together with students that don't know the tables, is not easy. But it is a tremendous challenge that I personally, after our group meetings, tackle in a completely different way.* (Sec 3)

Managing instruction

In the Prim-group the teachers are more concerned with assessment topics and the role that accounts-of in connection with the open-ended tasks plays, in giving information about the learning of individual students:

> *Short notes on every student provide a good starting point for the evaluative conversations with parents. In addition I feel certain about what every student understands and can do. (Prim 2)*

> *Before I always started a new topic with an exposition. Now I look for open tasks that start at what the students know, and that lead them into the new topic.* (Prim 2)

In the Sec-group the teachers focus more on the dominating tradition of skills and the exercises of these. Through their participation in the project this heavy tradition is beginning to losen up:

The endless exercising of skills have given away for … . My students now work more practically with math and above all more problem solving. (Sec 4)

Exercising of skills (which of course still must be there) were given away for other types of tasks, i.e. activities, thought provokers, and open tasks. (Sec 5)

Reported development

In their written evaluations the teachers in general are very positive about what they have learned during the project. The themes of these reports are both about their self-image as a teacher and about the role that student thinking plays during the lessons:

I now feel more secure as a math teacher, … and I think that my students enjoy their math lessons much more. What has changed the most is the content of the lessons. (Prim 6)

My way of looking at teaching math has changed. I listen more to the questions and ideas of my students. And I myself pose different questions now. (Prim 4)

I listen much more to what the students are thinking, how they tackle the problems. More time is spent on students expressing their thoughts, both to their peers and to me. (Sec 5)
As time went on I realised that the active lessons gave me and the students more than the textbook did. (Prim 7)

I have stopped looking for what the students have done … and still I have a better image of what the students know. (Prim 5)

What they express in their evaluations after the second and last year of the project is over all more positive than they did in the interviews after the first year. This can be interpreted as an indication of that the kind of learning processes that is aimed for in this project takes time and perserverance.

Group meetings

Those who mention the group meetings in their evaluations mostly do so by saying that the meetings were experienced as stimulating and inspiring for the development of individual participants. Some also mention the importance of an open and allowing atmosphere:

A possibility to discuss mathematics on a different level than it is usually done (Sec 4)

I have got ideas from the other members of the group that I have tested with my own students (Prim 3)

We tested different tasks, and we did it in different ways, experiences differed and gave the breadth and the depth. There were plenty of room during our discussions, we felt free to risk trying out ideas. (Prim 7)

The members of the project team were also reported as important for securing the depth in the discussions and for giving connections to research.

Conclusions and discussion

Firstly we will consider the witnessed effects of working with accounts-of. Although this tool sometimes is seen as rather cumbersome, especially for participants that does have some resistance to writing, it has worked productively as a way of gathering a kind of information that by Kazemi & Franke (2004) has been identified as essential for good in-service training. Many of the participating teachers witness directly or indirectly and in different forms about an increased awareness of student

thinking, and its importance for instruction. Effects of this kind are reported in most of the different themes identified in the results.

Being a part of the "social community" of a professional group that share the same kind of professional experiences stand forth as being very important. An essential part of this seems due to the way that the accounts-of are treated. The supervisors, i.e. the project leaders, are important in more than one way. They are responsible for assuring a climate of openness, where different views and experiences are received and treated respectfully. They are also important for bringing in new perspectives brought in from research and other areas of reflected experiences that ordinary teachers very seldom have the opportunity to meet. Similar results are reported from the CGI-project (Fennema e.a., 1996).

We now turn to the effects of working with open-ended tasks. Some of the participants report positive effects for issues of inclusion both in primary and in secondary classrooms. These findings are similar to the ones reported by Sullivan e.a. (2004) and Sullivan e.a. (2006), although not as systematically and ambitiously investigated.

At the same time as we report of positive and encouraging effects of the use of accounts-of to increase the awareness of teachers for student thinking and variability, and the effects of using more open-ended and investigative tasks, we must also make the conclusion that not all teachers learn as much as others. Since this kind of developmental project really is about teacher's learning how to deal with a new way of looking at the enterprise of mathematics teaching, it would be funny not to have an individual variation also in this learning process. We asked for participants that were willing to experiment with and to try out things in their mathematics teaching. The gross majority of the participants also met these criteria, but not all. Anyhow, change (learning) takes time. Our overall impression is that the participants needed the first year to come to terms with a new way of working and thinking about mathematics teaching and learning, and it was not until the second year that their grasp of the aims of the project and their reported teaching really started to change. So it seems that two years have been a necessity for the positive experiences that the participating teachers made To change your way of working, and to start building your teaching more on an increased awareness of student thinking and learning takes time and perseverance.

In conclusion accounts-of seems to have a good potential for developing teacher's awareness of student reasoning and thinking, i.e. giving them a better stance for following their learning, and that open-ended tasks do seem to have a potential for managing a better inclusive mathematics teaching.

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