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New good quasi-cyclic codes and codes with improved minimum distance

Research Article

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Abstract: One of the most important and challenging problems in coding theory is to construct codes with optimal parameters. As a generalization of cyclic codes, quasi-cyclic (QC) codes as well as quasi-twisted (QT) codes have been shown to contain record-breaking codes. In this paper, various computer algorithms have been used to search for good QC codes. A lot of good new QC codes have been found and they have been used to construct new linear codes. A total 11 new codes that improve the bound on the minimum distance are presented.

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1. Introduction

A linear $[n, k, d]_q$ code over finite field $GF(q)$ is a k-dimensional subspace of $GF(q)^n$, where n is the block length, k is the dimension of the code, and d is the minimum distance between any two different codewords. The minimum distance determines the error-correcting or error-detecting capability. A central and fundamental problem in coding theory is to find the optimal values of the parameters of a linear code and construct codes with these parameters. Grassl [16] maintains online code tables of linear codes for small block length and code dimension over small finite fields. The code tables contain both the lower bounds and upper bounds on the minimum distance. A code with a minimum distance meeting the upper bound is said to be optimal, while a code with a minimum distance meeting the lower bound is called best-known (since no other code with the same block length n, code dimension k, and with larger minimum distance is known). To construct codes with the best possible minimum distances is shown to be very difficult and challenging. For small code dimension and block length, it is possible to do exhaustive computer search. The problem becomes intractable when both the code dimension and block length become large. It has been shown that subclasses of linear codes with rich mathematical structures can be used to reduce the search time complexity. During the last decades, the classes of

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quasi-cyclic (QC) codes and quasi-twisted (QT) codes have been shown to contain many good codes, and many record-breaking QC/QT codes have been constructed [1–3, 5–9, 11–15, 17–23] A lot of codes that reach the lower bound on the minimum distance are QC/QT codes [16]. An online database of good QC/QT codes is available [10]. In this paper, various algorithms to search for good QC/QT codes have been applied, and lot of good new QC/QT codes have been obtained. By applying Construction X with new constructed QC codes, 5 new linear codes have been constructed. A total of 11 new linear codes that improve the lower bounds on the minimum distance have been presented in this paper.

2. Computer search for quasi-cyclic codes

A linear $[n,k,d]_q$ code C is called cyclic if a codeword $(a_0, a_1, \ldots, a_{n-1})$ is in C, then so is $(a_{n-1}, a_0, a_1, \ldots, a_{n-2})$. A code is said to be quasi-cyclic (QC) if a cyclic shift of any codeword by p positions is also a codeword. Therefore, a cyclic code is a QC code with $p = 1$. The length n of a QC code is a multiple of p, i.e., $n = pm$. A cyclic matrix is also called a circulant matrix. An $m \times m$ cyclic matrix is defined as

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & \ldots & a_{m-1} \\ a_{m-1} & a_0 & a_1 & \ldots & a_{m-2} \\ a_{m-2} & a_{m-1} & a_0 & \ldots & a_{m-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_1 & a_2 & a_3 & \ldots & a_0 \end{bmatrix}, \tag{1}$$

and the algebra of $m \times m$ cyclic matrices over $GF(q)$ is isomorphic to the algebra in the ring $GF(q)[x]/(x^m - 1)$, if A is mapped onto the polynomial formed by the elements of its first row, $a(x) = a_0 + a_1 x + \ldots + a_{m-1}x^{m-1}$, with the least significant coefficient on the left. The polynomial $a(x)$ is also called the defining polynomial of the matrix A.

The polynomials $bx^j a(x)$, where b is a non-zero element in $GF(q)$, and $j = 0, 1, 2, \ldots, m-1$, form an equivalent class, and they generate the equivalent cyclic codes. Therefore, it is enough to take one representative from each equivalent class. The number of nonzero representatives (used as defining polynomials) for $m \times m$ circulant matrices over $GF(q)$ is given below [23]:

$$b(m,q) = \frac{1}{(q-1)m} \sum_{d|m} \phi(d)(q^{\frac{m}{d}} - 1) \gcd(d, q-1), \tag{2}$$

where $\phi(d)$ is Euler's totient function.

The generator matrix of a QC code can be transformed into rows of $m \times m$ circulant matrices by suitable permutation of columns. An h-generator QC code has a generator matrix of the following form:

$$G = \begin{bmatrix} G_{1,1} & G_{1,2} & G_{1,3} & \ldots & G_{1,p} \\ G_{2,1} & G_{2,2} & G_{2,3} & \ldots & G_{2,p} \\ G_{3,1} & G_{3,2} & G_{3,3} & \ldots & G_{3,p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ G_{h,1} & G_{h,2} & G_{h,3} & \ldots & G_{h,p} \end{bmatrix}, \tag{3}$$

where $G_{i,j}$ are $m \times m$ circulant matrices, for $i = 1, 2, \ldots, h$, and $j = 1, 2, \ldots, p$. Let $g_{ij}(x)$ be the defining polynomial of the matrix $G_{i,j}$. Then the defining polynomials for the h-generator QC code with generator matrix given in (3) can be written as

$$(g_{11}(x), g_{12}(x), g_{13}(x), \ldots, g_{1p}(x), \ldots, g_{h1}(x), g_{h2}(x), g_{h3}(x), \ldots, g_{hp}(x)).$$

In Magma [4], the parameter h is called the height.

In the computer search algorithms presented in [8, 17–19], a weight matrix W is used in the computation of the minimum distance of a 1-generator QC code. The general $r \times s$ weight matrix has the following form:

$$W = \begin{bmatrix} w_{0,0} & w_{0,1} & \dots & h_{0,s-1} \\ w_{1,0} & w_{1,1} & \dots & h_{1,s-1} \\ \vdots & \vdots & \ddots & \vdots \\ w_{r-1,0} & w_{r-1,1} & \dots & w_{r-1,s-1} \end{bmatrix}, \tag{4}$$

where the entry $w_{i,j}$ is the Hamming weight of $I_i(x)g_j(x) \mod x^m - 1$, $I_i(x)$ is the i-th distinct information polynomial after the equivalent reduction, and $g_j(x)$ is the j-th defining polynomial [17–20, 23]. With this weight matrix, to construct a best QC $[pm, k]$ code, it is sufficient to find p columns that give the maximum of minimum row sums.

In practical implementation of search algorithms, the computer storage is limited. When the code dimension becomes large, the number of defining polynomials would be too large, which makes the weight matrix too large for a general computer to complete the search in reasonable time. In [6], defining polynomials of specific weights were selected, while in [8, 9], a specified number of randomly chosen defining polynomials were selected. For example, the number of defining polynomials for $m = 88$, $k = 18$ and $q = 3$ is 2204293. It is too large to store a weight matrix of 2204293×2204293 inside the computer memory. If 100 randomly chosen defining polynomials are used, then the weight matrix is reduced to the size of 2204293×100, which is possible in most laptop or desktop computers. Of course, the selection of the number of defining polynomials during the search would limit how good a code can be found. But the experience shows that even 100 randomly chosen defining polynomials are used, many good QC codes can be found. For example, by applying the limited search algorithms, a new QC $[176, 18, 88]_3$ code is found, which improves the bound on minimum distance.

3. The new good and improved codes

For a given size m of the circulant matrix and code dimension k, first the non-equivalent defining polynomials and distinct information polynomials were calculated [17, 19]. Then 100 defining polynomials are selected randomly to compute the weight matrix. For small p ($p = 2$ or 3), an exhaustive search among these polynomials is taken, otherwise the iterative search algorithm is applied [8]. Via the computer search, more than 300 good QC codes have been obtained. These codes are included in online database of quasi-twisted codes [10]. For example, for $m = 11$, all best-known QC $[pm, 10]_7$ codes with $p = 2, \dots, 9$ have been found, as shown in Table 1. The details of the codes can be found in the online database [10]. In the rest of this paper, the codes that improve the minimum distances in [16] are presented.

Table 1. Best-known $[pm, 10, d]_7$ codes with $m = 11$.

p	n	k	d	reference
2	22	10	10	[10]
3	33	10	18	[16]
4	44	10	26	[10]
5	55	10	34	[16]
6	66	10	42	[16]
7	77	10	50	[10]
8	88	10	59	[10]
9	99	10	68	[10]

Theorem 3.1. *There exist QC* $[93, 9, 62]_5$, $[75, 11, 45]_5$, *and* $[176, 18, 88]_3$ *codes.*

Proof. The QC $[93, 9, 62]_5$ code is constructed with $m = 31$, and its defining polynomials are $g_1(x) = x^{27} + x^{26} + 2x^{25} + x^{24} + 2x^{23} + 4x^{22} + 2x^{21} + 4x^{20} + 3x^{19} + 4x^{18} + 3x^{16} + x^{13} + x^{11} + x^{10} + 4x^8 + 3x^7 + 4x^6 + 4x^5 + 3x^4 + 3x + 4$, $g_2(x) = x^{27} + x^{24} + 3x^{23} + 2x^{21} + 2x^{20} + 2x^{19} + 2x^{18} + x^{17} + 3x^{15} + 3x14 + x^{13} + 2x^{12} + x^{11} + 2x^{10} + 2x^9 + 3x^8 + 4x^7 + 3x^6 + 2x^5 + 2x^4 + x^3 + 2x^2 + 3x + 2$, and $g_3(x) = x^{29} + 3x^{28} + 4x^{26} + x^{25} + x^{24} + 3x^{22} + 4x^{21} + 3x^{20} + x^{19} + 4x^{18} + 2x^{17} + x^{16} + 3x^{15} + 3x^{14} + 3x^{13} + 2x^{12} + x^{11} + x^{10} + 3x^9 + 3x^8 + x^7 + 2x^6 + x^5 + 2x^4 + x^3 + 3x^2 + x + 2$.

The QC $[75, 11, 45]_5$ code is constructed with $m = 15$, and its defining polynomials are $g_1(x) = x^{14} + x^{13} + 3x^{12} + 4x^{11} + x^{10} + x^9 + 4x^8 + x^7 + 3x^6 + 2x^5 + x^4 + 3x^3 + x^2 + 3x + 2$, $g_2(x) = x^{12} + 3x^{11} + 4x^{10} + 3x^9 + 4x^8 + 4x^7 + 2x^5 + 3x^4 + 4x^3 + 3x^2 + x + 4$, $g_3(x) = x^{13} + 2x^{12} + 3x^{10} + x^9 + 4x^8 + 4x^6 + 3x^5 + x^3 + 4x^2 + 2x + 3$, $g_4(x) = x^{13} + x^{12} + 3x^{10} + 4x^9 + 3x^8 + 3x^7 + x^6 + x^5 + 3x^3 + 2x^2 + 4x + 2$, and $g_5(x) = x^{11} + 4x^{10} + x^9 + 2x^8 + 2x^7 + x^5 + 2x^4 + x^2 + 2x + 4$.

The QC $[176, 18, 88]_3$ code is constructed with $m = 88$ and its defining polynomials are $g_1(x) = x^{83} + 2x^{82} + 2x^{81} + x^{80} + 2x^{79} + 2x^{77} + x^{76} + x^{75} + x^{74} + 2x^{73} + x^{72} + x^{70} + 2x^{69} + 2x^{68} + 2x^{63} + 2x^{62} + 2x^{60} + 2x^{59} + x^{58} + 2x^{56} + x^{55} + x^{54} + x^{53} + x^{52} + x^{51} + x^{50} + x^{47} + x^{46} + 2x^{45} + 2x^{43} + 2x^{42} + 2x^{41} + 2x^{40} + 2x^{37} + 2x^{36} + x^{35} + x^{34} + x^{33} + 2x^{32} + 2x^{30} + 2x^{29} + x^{28} + x^{27} + x^{26} + 2x^{24} + x^{22} + 2x^{21} + 2x^{20} + x^{17} + 2x^{15} + 2x^{14} + 2x^{12} + 2x^{11} + 2x^{10} + 2x^8 + x^6 + x^5 + 2x^4 + x^3 + x + 1$, and $g_2(x) = x^{83} + x^{81} + x^{79} + 2x^{78} + 2x^{77} + 2x^{76} + 2x^{75} + x^{73} + x^{72} + 2x^{71} + 2x^{70} + 2x^{69} + x^{68} + x^{67} + x^{64} + 2x^{63} + 2x^{62} + 2x^{61} + 2x^{59} + x^{58} + 2x^{57} + x^{55} + 2x^{54} + x^{51} + 2x^{49} + 2x^{47} + 2x^{46} + 2x^{45} + x^{44} + 2x^{42} + 2x^{41} + x^{39} + 2x^{38} + x^{37} + 2x^{35} + x^{34} + x^{32} + 2x^{30} + 2x^{29} + x^{28} + x^{25} + 2x^{24} + 2x^{23} + 2x^{22} + x^{20} + x^{18} + 2x^{16} + 2x^{14} + x^{13} + x^{11} + x^{10} + 2x^9 + 2x^8 + 2x^7 + 2x^6 + 2x^4 + x^3 + x^2 + x + 1$. □

All the codes have been checked in Magma algebraic system [4] [4] and their weight distributions of these codes can be found in online database of quasi-twisted codes [10]. It should be noted, that a new $[176, 18, 89]_3$ code was found after our codes were reported [16], and it was based on our reported $[176, 17, 90]_3$ code as given in Theorem 3.2 below.

Construction X is a method to construct new codes by combining 3 existing codes. Let $C_1 = [n, k_1, d_1]_q$ and $C_2 = [n, k_2, d_2]_q$ be a pair of nested codes, where $C_1 \subset C_2$. Let $C_3 = [n_3, k_2 - k_1, d_3]_q$ be an auxiliary code. Then there exists a $C = [n + n_3, k_2, d]_q$ code with $d \geq \min(d_1, d_2 + d_3)$.

Theorem 3.2. *There exists* $[39, 10, 21]_5$, $[73, 9, 47]_5$, $[68, 11, 40]_5$, $[178, 18, 90]_3$, *and* $[119, 30, 34]_2$ *codes.*

Proof. Let C_1 be the QC $[38, 10, 20]_5$ code with $m = 19$. Its defining polynomials are $g_1(x) = x^{15} + x^{14} + 4x^{13} + x^{12} + x^{11} + 3x^{10} + 3x^9 + x^7 + 2x^6 + x^5 + 4x^4 + 2x^3 + 4x^2 + 4$, and $g_2(x) = x^{16} + 2x^{15} + 4x^{14} + x^{13} + 4x^{12} + x^{11} + x^9 + 3x^8 + x^6 + 2x^5 + 3x^4 + 3x^3 + 4x^2 + 4$. Let C_2 be the QC $[38, 9, 21]_5$ code with $m = 19$. Its defining polynomials are $g_1(x) = x^{16} + 3x^{14} + 2x^{13} + 2x^{11} + 2x^9 + x^8 + x^7 + 4x^6 + 3x^5 + 3x^4 + 2x^3 + x^2 + 4x + 1$, and $g_2(x) = x^{17} + x^{16} + 2x^{15} + 2x^{14} + 3x^{13} + 2x^{12} + 4x^{11} + x^{10} + 2x^9 + 2x^8 + x^7 + x^6 + x^5 + x^3 + x^2 + 4x + 1$. Let C_3 be an $[1, 1, 1]_5$ code. By applying Construction X, the new $[39, 10, 21]_5$ code can be constructed.

Let C_1 be the QC $[72, 9, 46]_5$ code with $m = 24$. Its defining polynomials are $g_1(x) = x^{22} + 4x^{20} + 3x^{19} + 2x^{18} + x^{17} + x^{16} + 2x^{15} + 3x^{14} + 4x^{13} + 2x^{12} + 2x^{10} + 3x^8 + 3x^7 + 4x^6 + 2x^4 + 2x^3 + 4x^2 + 3x + 4$, $g_2(x) = x^{20} + 3x^{19} + x^{18} + 2x^{17} + x^{16} + 3x^{15} + x^{14} + 2x^{13} + x^{12} + x^{11} + 4x^9 + 2x^8 + 4x^7 + 3x^6 + 2x^5 + 3x^4 + 3x^3 + 3x^2 + 3x + 2$, and $g_3(x) = x^{21} + 4x^{20} + 4x^{18} + 4x^{17} + 3x^{16} + 4x^{15} + 4x^{14} + x^{13} + 4x^{12} + x^{11} + 3x^{10} + 4x^9 + 4x^8 + 2x^7 + x^6 + 3x^5 + x^4 + 3x^2 + 2x + 2$. Let C_2 be the QC $[72, 8, 47]_5$ code with $m = 24$. Its defining polynomials are $g_1(x) = x^{23} + 3x^{22} + 4x^{21} + x^{19} + 2x^{18} + 4x^{17} + 4x^{15} + 3x^{14} + 4x^{13} + x^{12} + 2x^{11} + x^{10} + 3x^9 + 2x^8 + 3x^7 + 2x^6 + 2x^5 + 3x^4 + 3x + 2$, $g_2(x) = x^{21} + x^{20} + 2x^{17} + x^{16} + 2x^{13} + 4x^{12} + 3x^{11} + 4x^{10} + 4x^9 + x^6 + 4x^5 + 2x^4 + 2x^3 + 2x^2 + x + 1$, and $g_3(x) = x^{22} + 2x^{21} + 2x^{20} + 4x^{19} + x^{18} + 3x^{16} + x^{15} + 3x^{14} + 2x^{13} + 3x^{12} + x^{11} + 3x^{10} + x^9 + 4x^8 + 2x^7 + x^6 + 3x^4 + 3x^3 + x^2 + 3x + 1$. Let C_3 be an $[1, 1, 1]_5$ code. By applying Construction X, the new $[73, 9, 47]_5$ code can be constructed.

Let C_1 be the QC $[66, 11, 38]_5$ code with $m = 22$. Its defining polynomials are $g_1(x) = x^{19} + x^{18} + 4x^{17} + 4x^{16} + x^{15} + 3x^{14} + 3x^{13} + 2x^{11} + 2x^9 + 4x^8 + 4x^7 + x^6 + 2x^5 + 2x^4 + x^2 + 4x + 2$, $g_2(x) = x^{19} + x^{18} + 4x^{17} + x^{15} + 4x^{14} + 3x^{13} + 2x^{12} + 4x^{11} + x^{10} + 3x^9 + 4x^7 + 2x^6 + 4x^5 + 3x^4 + x^3 + 4x + 1$, and $g_3(x) = x^{18} + 4x^{17} + 2x^{16} + 3x^{15} + 3x^{14} + x^{13} + 2x^{11} + 4x^{10} + 3x^9 + 2x^5 + 4x^2 + 1$. Let C_2 be the QC $[66, 10, 40]_5$ code with $m = 22$. Its defining polynomials are $g_1(x) = x^{20} + 3x^{18} + 2x^{16} + 2x^{15} +$

$2x^{13} + 2x^{12} + 3x^{11} + 2x^{10} + 2x^9 + 2x^7 + x^6 + 3x^4 + x^3 + 3x^2 + 3x + 3$, $g_2(x) = x^{20} + 3x^{18} + x^{17} + x^{16} + 3x^{15} + 4x^{14} + 4x^{13} + 2x^{12} + 2x^{11} + 2x^{10} + 2x^9 + 4x^8 + 3x^7 + 2x^6 + 4x^5 + 3x^4 + 4x^3 + 4x^2 + 2x + 4$, and $g_3(x) = x^{19} + 3x^{18} + 3x^{17} + x^{16} + 3x^{14} + 4x^{13} + 2x^{12} + 2x^{11} + 4x^{10} + 2x^9 + 2x^6 + 3x^5 + 4x^3 + x^2 + x + 4$. Let C_3 be an $[2, 1, 2]_5$ code. By applying Construction X, a new $[68, 11, 40]_5$ code can be constructed.

Let C_1 be the QC $[176, 17, 90]_3$ code with $m = 88$. Its defining polynomials are $g_1(x) = x^{84} + x^{82} + 2x^{79} + 2x^{78} + 2x^{76} + 2x^{75} + x^{72} + x^{71} + x^{69} + 2x^{68} + 2x^{64} + x^{63} + 2x^{62} + 2x^{61} + x^{60} + x^{58} + 2x^{57} + 2x^{55} + 2x^{54} + 2x^{53} + 2x^{52} + 2x^{51} + x^{50} + x^{48} + 2x^{47} + 2x^{45} + 2x^{44} + x^{43} + x^{42} + x^{41} + 2x^{40} + 2x^{38} + x^{37} + 2x^{35} + 2x^{34} + 2x^{32} + 2x^{31} + x^{30} + 2x^{28} + 2x^{27} + x^{26} + 2x^{25} + 2x^{24} + x^{23} + x^{21} + 2x^{20} + x^{18} + x^{17} + 2x^{16} + x^{15} + 2x^{14} + 2x^{13} + x^{12} + x^{11} + 2x^{10} + 2x^9 + 2x^8 + x^7 + 2x^6 + x^3 + x^2 + 2x + 1$, and $g_2(x) = x^{84} + x^{83} + x^{82} + x^{81} + x^{80} + x^{78} + x^{77} + x^{76} + 2x^{75} + x^{74} + 2x^{73} + x^{71} + x^{70} + 2x^{68} + x^{67} + x^{65} + x^{63} + x^{62} + 2x^{61} + 2x^{60} + 2x^{57} + x^{56} + 2x^{54} + x^{52} + x^{51} + 2x^{50} + 2x^{49} + 2x^{48} + x^{47} + x^{46} + x^{44} + 2x^{43} + x^{42} + 2x^{41} + x^{40} + x^{37} + 2x^{36} + x^{34} + x^{33} + x^{32} + 2x^{31} + x^{30} + x^{28} + x^{26} + x^{24} + x^{23} + 2x^{22} + x^{21} + x^{20} + x^{19} + x^{18} + 2x^{17} + 2x^{16} + 2x^{15} + x^{13} + x^{12} + 2x^{11} + x^9 + x^8 + x^7 + 2x^6 + 2x^5 + 2x^3 + 2x^2 + 2x + 1$. Let C_2 be the QC $[176, 18, 88]_3$ code given above. Let C_3 be the $[2, 1, 2]_3$ code. By applying Construction X, a new $[178, 18, 90]_3$ can be constructed.

Let C_1 be the 3-generator QC $[116, 30, 32]_2$ code with $m = 29$. Its defining polynomials are $g_1(x) = x + 1$, $g_2(x) = x^{12} + x^{11} + x^9 + x^6 + x^5 + x^3 + x^2 + 1$, $g_3(x) = x^{24} + x^{23} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{12} + x^9 + x^7 + x^5 + x^4 + x^3 + x^2 + x + 1$, $g_4(x) = x^{20} + x^{19} + x^{17} + x^{15} + x^9 + x^4 + x + 1$, $g_5(x) = g_7(x) = g_10(x) = g_12(x) = x^{28} + x^{27} + x^{26} + x^{25} + x^{24} + x^{23} + x^{22} + x^{21} + x^{20} + x^{19} + x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{11} + x^{10} + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$, and $g_6(x) = g_8(x) = g_9(x) = g_11(x) = 0$. Let C_2 be the QC $[116, 28, 34]_2$ code with $m = 29$. Its defining polynomials are $g_1(x), g_2(x), g_3(x)$ and $g_4(x)$ as given in the $[116, 30, 32]_2$ code above. Let C_3 be an $[3, 2, 2]_2$ code. By applying Construction X, a new $[119, 30, 34]_2$ code can be constructed. □

All the codes given above improve the minimum distances in [16]. By applying puncturing method, new improved $[92, 9, 61]_5, [67, 11, 39]_5, [118, 30, 33]_2$, and $[177, 18, 89]_3$ codes are obtained. All the codes given in the paper have been checked with the Magma algebraic system [4] and included in [16] now.

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